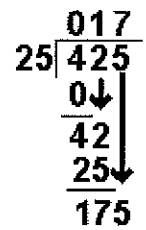
APES MATH TIPS for the AP Exam

Increasingly, students are asked to demonstrate their sense of math by calculating their answers by hand and showing work instead of using a calculator. Numbers lose their meaning too often when students become completely calculator-dependent. Practice!

1) **Show all *work*.** No work, no credit.
2) **Show all *units*.** Units provide valuable information.
3) **Be proficient at *unit manipulation*,** also called dimensional analysis or factor label. This is one of the most important math skills, because you will have to fit numbers with units together through multiplication and division to get the desired results.
4) ***Add, subtract, multiply,* and *divide* numbers without a calculator.** Multiplication and division are usually seen more than addition and subtraction. The math is able to be done without a calculator, but because students use calculators so much, even advanced students can be awkward when doing long division by hand. Watch the proper placement of the numbers. For 425/25, see the setup from www.mathisfun.com →



5) **Develop good *"math sense"* or *"math literacy."*** The answers should make sense. If you calculate a cost of \$50 billion per gallon of water, does this seem right?
6) **Know *simple conversion factors*** such as the number of days in a year or hours in a day. Other numbers to know: U.S. population & World population
7) **Know and convert *metric prefixes*.**

T	tera-	10^{12}	(trillion 1,000,000,000,000)
G	giga-	10^{9}	(billion 1,000,000,000)
M	mega-	10^{6}	(million 1,000,000)
k	kilo-	10^{3}	(1000)
h	hecto-	10^{2}	(100)
da	deka-	10^{1}	(10)
d	deci-	10^{-1}	(0.1)
c	centi-	10^{-2}	(0.01)
m	milli-	10^{-3}	(0.001)
μ	micro-	10^{-6}	(one-millionth 0.000001)
n	nano-	10^{-9}	(one-billionth 0.000000001)

8) Understand common statistical terms. The **mean** is the mathematical average. The **median** is the 50^{th} percentile, which is the middle value in the distribution of numbers when ranked in increasing order. The **mode** is the number that occurs most frequently in the distribution.
9) **Be comfortable working with *negative numbers*.** Going from -8 °C to +2 °C is a 10° change.
10) **Recognize units of *area* and *volume*, and be able to *convert volumes*.**
    1 m = ____ mm… answer → 1000
    $1\ m^3$ = ____ mm^3 answer → $1^3\ m^3 = 1000^3\ mm^3 \quad (10^3)^3 = 10^9\ mm^3$

For area conversions, square the number, square the unit. For volume conversions, cube the number, cube the unit.

11) **Calculate *percentages*.** Example: 80/200 = 40/100 = 0.4 = 40%

12) **Put very large or very small numbers into *scientific notation*.**
$310{,}000{,}000 = 310$ million $= 310 \times 10^6 = 3.1 \times 10^8$
$0.000\,000\,000\,000\,097 = 9.7 \times 10^{-14}$

13) **Work *scientific notation problems* without a calculator.** Multiplication and division will be common. Multiplying numbers in scientific notation requires the exponents to be added. Dividing numbers in scientific notation requires exponents to be subtracted.

14) **Know growth rate calculations. Growth rate = [CRUDE BIRTH RATE + immigration)] – [(CRUDE DEATH RATE + emigration)]** (see 2003 FRQ #2)
CBR = crude birth rate = *# births per 1000, per year*
CDR = crude death rate = *# deaths per 1000, per year*
(CBR – CDR) / 10 = percent change

15) **Calculate percent change:**
a) The rate of change (**percent change**, growth rate) from one period to another =
$[(V_{present} - V_{past}) / V_{past}] * 100$ (where V = value)
b) **Annual rate of change:** take answer from step a) and divide by the number of years between past and present values

Example: A particular city has a population of 800,000 in 1990 and a population of 1,500,000 in 2008. Find the growth rate of the population in this city:

Growth Rate = (1,500,000 - 800,000) / 800,000 * 100 = 700,000/800,000 * 100 = 87.5%
Average Annual Growth Rate = 87.5% / 18 years = 4.86 %

16) **Calculate percent difference.**

$$\textbf{Percentage Difference} = \left| \frac{\textbf{First Value - Second Value}}{\textbf{(First Value + Second Value) / 2}} \right| \times 100\%$$

17) **Know the *Rule of 70* to predict doubling time.**
Doubling time = 70 / annual growth rate (in %, not decimal!) Example: If a population is growing at a rate of 4%, the population will double in 17.5 years. (70 / 4 = 17.5)

18) **Calculate *half-life*.**
AMOUNT REMAINING = (ORIGINAL AMOUNT)(0.5)^x
where x = number of half-lives **x = time / half-life**

19) **Calculate pH using –log [H^+].** $\log_{10} x = y$ and $10^y = x$.
Any pH problems are easily solved without a calculator. Remember that for every one-increment change in pH, the ions change by a factor of 10.
Example: If [H^+] is 10^{-6} M, the pH is 6 and the solution is a weak acid.

20) **Know that "*per capita*" means per person; per unit of population.**

21) **Graphing tips:** include a title and key; set consistent increments for axes; connect dots; interpolate and extrapolate; be comfortable with graphing by hand.

Tips for Solving Math Problems in APES

Energy Efficiency

Efficiency = (Work Output/Work Input) x 100%

Efficiency refers to the ability of an energy conversion reaction to convert energy from one form to the next without losing too much heat to the environment (electrical to light, chemical to mechanical, and etc.)

An energy conversion that doesn't lose any heat at all would have a100% efficiency rating.

If you have to determine the work input of a reaction and you are given the energy efficiency and work output, then divide the output by the efficiency percentage.

Dimensional Analysis Review

1) What units of measure do you want in the answer?

In this problem you want to know how many BTU's are needed each day. After you figure out what units you want to know, translate the English into Math. In Math terms, what you want to know is:

$$\frac{\text{BTU's}}{\text{Day}}$$

2) What do you know?

What do you know about "BTU's" or "days"? You know that there are 12, 000, 000 kWh produced in a day (this will be provided in a problem). You also know that if 1kWh is produced by 10, 000 BTU's. Connecting kWh from the two equivalencies, bridges the information we know to the information requested in the problem. When you have this kind of connection between units, then you know enough to solve the problem – but first translate what you know into math terms that you can use when solving the problem.

$$\frac{12,000,000\text{ kWh}}{\text{Day}} \quad \text{or} \quad \frac{\text{day}}{12,000,000\text{ kWh}} \quad \text{or} \quad \frac{1\text{ kWh}}{10,000\text{ BTU}} \quad \text{or} \quad \frac{10,000\text{ BTU}}{1\text{ kWh}}$$

3) Pick from these statements the ones that you actually need.

The trick is to pick the equivalencies that will cancel out the unit you don't want. You want "BTU's" on top in your answer. On the bottom you want "days". You need to get rid of the "kWh". You cancel "kWh" out by picking a factor that has "kWh" on top and one that has "kWh" on the bottom. This allows this unit to be cancelled out.

$$\frac{12,000,000\text{ kWh}}{\text{Day}} \times \frac{10,000\text{ BTU}}{1\text{ kWh}}$$

4. Solve it.

When you have cancelled out the units you don't want and are left only with the units you do want, then you know it's time to multiply all the top numbers together, and divide by all the bottom numbers.

$$\frac{12,000,000\text{ }\cancel{\text{kWh}}}{\text{Day}} \times \frac{10,000\text{ BTU}}{1\text{ }\cancel{\text{kWh}}}$$

Scientific Notation

1. Converting to Scientific Notation:

$0.00234 = 2.34 \times 10^{-3}$

Write the number .00234 as a coefficient. Coefficients need to be between 1 and 10. The coefficient is then multiplied by ten raised by an exponent. The number of places to the left that you move the decimal point is the exponent. If you move the decimal to the right, the exponent will be negative.

$= 0.0234/10$

$= 0.234/(10 \times 10)$

$= 2.34/(10 \times 10 \times 10)$

$= 2.34/(10^3)$

$= 2.34 \times 10^{-3}$

2. <u>Adding and subtracting two numbers written in scientific notation:</u>

$5.2 \times 10^3 + 3.6 \times 10^4 = 4.12 \times 10^4$

Factor out one of the powers of ten; usually the smaller one is the easiest. Divide both numbers by the power of ten and multiplying the whole quantity by the same power of ten. To divide one power of ten by another, simply subtract the two exponents. Next, convert the two numbers from scientific notation to real numbers. Now add the two numbers normally. Finally convert to scientific notation if the coefficient is less than 1 or greater than 10.

$5.2 \times 10^3 + 3.6 \times 10^4$
$= (5.2 \times 10^3/10^3 + 3.6 \times 10^4/10^3) \times 10^3$
$= (5.2 \times 10^0 + 3.6 \times 10^1) \times 10^3$
$= (5.2 + 36) \times 10^3$
$= 41.2 \times 10^3$
$= 4.12 \times 10^4$

$3.9 \times 10^{-6} - 6.9 \times 10^{-5} = -6.51 \times 10^{-5}$

Factor out one of the powers of ten. Next, convert both scientific notation numbers to real numbers. Subtract the two numbers normally and convert to scientific notation if the coefficient is not between 1 and 10 (or -1 and -10).

$3.9 \times 10^{-6} - 6.9 \times 10^{-5}$
$= (3.9 \times 10^{-6}/10^{-5} - 6.9 \times 10^{-5}/10^{-5}) \times 10^{-5}$
$= (3.9 \times 10^{-1} - 6.9 \times 10^0) \times 10^{-5}$
$= (.39 - 6.9) \times 10^{-5}$
$= -6.51 \times 10^{-5}$

3. <u>Multiplying two numbers written in scientific notation:</u>

$(4 \times 10^{-2}) \times (2 \times 10^{10}) = 8 \times 10^8$

Multiply the two coefficients and then multiply the two powers of ten by adding their exponents: since -2 + 10 = 8, then $10^{-2} \times 10^8$. Finally, combine your two answers and convert to scientific notation: 8×10^8.

$(4 \times 10^{-2}) \times (2 \times 10^{10})$
$= (4 \times 2) \times (10^{-2} \times 10^{10}$[add])
$= (8) \times (10^8)$
$= 8 \times 10^8$

4. <u>Divide two numbers written in scientific notation:</u>

$(4.2 \times 10^{-6}) / (6 \times 10^{-2}) = 7 \times 10^{-5}$

Divide the two coefficients: 4.2/6 = 0.7. Then, divide the two powers of ten by subtracting their exponents: Since -6 – (-2) = -6 + 2 = -4, then $10^{-6}/10^{-2} = 10^{-4}$. Finally, combine your two answers and convert to scientific notation.

$(4.2 \times 10^{-6}) / (6 \times 10^{-2})$
$= (4.2/6) \times 10^{-6}/10^{-2}$ [subtract])
$= (0.7) \times (10^{-4})$
$= 7 \times 10^{-5}$

Population Growth Rate

Growth rate is calculated by the change in the population divided by the total population, then multiplied by 100 to report the change in a percentage.

When given the CBR and the CDR use the following equation: $\frac{CBR - CDR}{10} = \%$

Rule of 70

Calculates the doubling time based upon a percentage.

$\frac{70}{\%\ \text{growth (do not convert to a decimal)}} = \text{years to double}$

Per capita

Data given divided by total population