

Answer  
key

# POPULATION CALCULATION WORKSHEET

You will need to be familiar with these equations.

## POPULATION DENSITY

$$\left( \frac{\text{population}}{\text{area}} \right) = \text{Population Density}$$

for example:  $\left( \frac{270,000,000 \text{ people}}{9,166,605 \text{ sq. km.}} \right) = 29 \text{ people per square kilometer}$

## BIRTH OR DEATH RATES:

$$\left( \frac{\text{\# of births or deaths per year}}{\text{Total population.}} \right) = \text{Birth or Death Rate}$$

*NOTE: to find Crude Birth/Death Rates, multiply the rate by 1,000*

for example:  $\left( \frac{23,452 \text{ births}}{942,721 \text{ people}} \right) = 0.025 = 2.5\% \text{ birth rate}$   
25 = Crude Birth Rate.

## FINDING POPULATION GROWTH RATE (r):

*(This does not include immigration or emigration)*

$$\left( \frac{\text{crude births} - \text{crude deaths}}{10} \right) = r \%$$

for example:  $\left( \frac{40 - 30}{10} \right) = 1.0\%$

## FINDING THE DOUBLING TIME OF A POPULATION: THE RULE OF 70!!!

*(This only applies if the population is growing exponentially)*

*Why 70? It is  $100 \times \ln(2)$ . What does that mean? Who cares...the math works!*

$$\left( \frac{70\%}{r \text{ (in percent form)}} \right) \text{ or } \left( \frac{0.7}{r \text{ (in decimal form)}} \right) = \text{Doubling Time (dt) in years.}$$

for example:  $\left( \frac{70\%}{7\%} \right) \text{ or } \left( \frac{0.7}{0.07} \right) = 10 \text{ years}$

## FINDING FUTURE POPULATION FROM GROWTH RATE:

$$(\text{initial population}) \times (\text{growth rate})^{\text{years}} = \text{Final Population}$$

*NOTE: a growth rate of 3% is expressed as 1.03; a growth rate of 0.25% is 1.0025*

for example:  $(468,843 \text{ people}) \times (1.03)^{10 \text{ years}} = 630,085 \text{ people}$

## Population Problems – SHOW ALL WORK!!

Given the following information, answer questions 1-4.

Schuhlsville is an island of 5000 square miles off the coast of Jabooty. There are currently 250,000 inhabitants of the island. Last year, there were 12,000 new children born and 10,000 people were recorded as deceased.

1. What is the current population density?

$$\frac{250,000 \text{ ppl}}{5000 \text{ mi}^2} = 50 \text{ ppl/mi}^2$$

2. What are the birth and death rates?

$$\text{BR} = \frac{12,000}{250,000} = 0.048 = 4.8\%$$

$$\text{DR} = \frac{10,000}{250,000} = 0.04 = 4\%$$

3. What is the population growth rate (r)?

$$4.8\% - 4\% = 0.8\%$$

4. In how many years will the population of Schuhlsville double?

$$\frac{70}{0.8} = 87.5 \text{ years}$$

Given the following information, answer questions 5-8.

The country of Transylvania contains 2.3 million people (vampires not included) and covers 800,000 square miles. In the year after the last census, there were 109,000 new children born and 111,000 people died.

5. What is the current population density?

$$\frac{2.3 \times 10^6 \text{ ppl}}{8 \times 10^5 \text{ mi}^2} = 2.9 \text{ ppl/mi}^2$$

6. What are the birth and death rates?

$$\text{BR} = \frac{109,000}{2.3 \times 10^6} = 0.047 = 4.7\%$$

$$\text{DR} = \frac{111,000}{2.3 \times 10^6} = 0.048 = 4.8\%$$

7. What is the population growth rate (r)?

$$4.7\% - 4.8\% = -0.1\%$$

8. In how many years will the population of Transylvania double?

$$\frac{70}{-0.1} = 700 \text{ years}$$

9. Given a 2010 world population growth rate of about 1.3% per year, how long would it take the world's population to double?

$$\frac{70}{1.3} = 54 \text{ years}$$

How old will you be when this doubling occurs?

$$21 + 54 = 75$$

10. If a country doubles its population in 56 years, what was its population growth rate during that time?

$$\frac{70}{x} = 56 \rightarrow \frac{70}{56} = x = 1.25\%$$

11. Calculate the growth rates and doubling times for the countries listed below.

Country	Birth Rate (2011)	Death Rate (2011)	Growth Rate (r)	Doubling Time
United States	13	8	$\frac{13-8}{10} = .5$	140
Mexico	19	5	$\frac{19-5}{10} = 1.4$	50
Japan	8	9	$-.1$	700
United Kingdom	13	9	$.4$	175
China	12	7	$.5$	140
India	23	7	$1.6$	43.8
Nigeria	41	16	$2.5$	28
South Africa	21	14	$.7$	100
Canada	11	7	$.4$	175
Italy	9	10	$-.1$	700

12. According to the 2010 census, Cedar Rapids contained 126,326 people. In 2011, there were an estimated 127,904 people. That translates to a growth rate of 1.2%. Based on this growth rate, what will the population of Cedar Rapids be 5 years from now?

\*Need calc.

$$127,904 \cdot (1.012^5) = 135,764 \text{ ppl}$$

...10 years from now?

$$127,904 \cdot (1.012^{10}) = 144,108 \text{ ppl}$$

...50 years from now?

$$127,904 \cdot (1.012^{50}) = 232,225 \text{ ppl}$$

...100 years from now?

$$127,904 \cdot (1.012^{100}) = 421,634 \text{ ppl}$$

13. In April of 2010 the U.S. population was 308,745,538 and it is growing by about .97%. Assuming a constant growth rate, what will the population be in 2020?

\*Need Calc.

$$308,745,538 (1.0097^{10}) = 340,035,494$$

...in 2050?

$$308,745,538 (1.0097^{40}) = 454,250,005$$

...in 2100?

$$308,745,538 (1.0097^{90}) = 736,059,213$$

14. What would happen to the population growth rate of a country that maintains a high crude birth rate of 32 but was able to reduce their crude death rate from 28 to 12?

$$BR \frac{32-28}{10} = .4\%$$

$$DR \frac{32-12}{10} = 2\%$$

What would happen to the doubling time of this country?

$$\frac{70}{1.4} = 50 \text{ yrs}$$

$$\frac{70}{2} = 35 \text{ yrs}$$

$$\frac{70}{1.0} = 70 \text{ yrs}$$

15. We are currently adding 84 million people to the world's population each year. That is about 229,000 each day. Below is a listing of some of the world's worst disaster, along with an approximate death toll. At today's growth rate, determine how many minutes, hours, days, weeks, or months it would take to replace those lost.

Past disasters	Approximate # of deaths	Present world population growth replaces this # in what time span?
Hurricane Katrina	1836	11.54 min
September 11, 2001 attacks	2996	18.8 min
U.S. accidental deaths in 2007	123,700	12.96 h
Sumatra tsunami on 12/26/04	225,000	23.6 h
American deaths in all wars as of 2010	655,000	2.86 d
Total U.S. auto deaths through 2007	3,000,000	13.1 d
Influenza epidemic, 1918	21,000,000	91.7 d
Total AID's deaths through 2005	25,000,000	109.2 d
The Black Plague, 1347-51	75,000,000	327.5 d

## Population Math Practice – How do I do this?!

Please put this handout in your notebook and solve the problems there!

Don't use a calculator... you may estimate!

How do I calculate a population's growth rate?  $\text{Growth rate} = \frac{[b - d] + [i - e]}{\text{Total Population}} \times 100 = \underline{\hspace{2cm}}\%$

How do I calculate a population's net annual percentage growth?  $\frac{\text{Growth rate}}{\text{Total Population}} \times 100 = \underline{\hspace{2cm}}\%$

How do I determine how long it would take a population to double? Rule of 70 (see intro to APES notes)

How do I determine population growth?

Brentwood has a population of 55,000. Let's say you have an annual growth rate of 10%. You want to know what the new population of Brentwood will be after 1 year.

Current population x Growth Rate (in decimal form) = number of people being added

Current population + number of people being added = New population

(If you wanted to know for two years... you'd have to go through the math twice!)

1. If a city of population 10,000 experiences 100 births, 40 deaths, 10 immigrants, and 30 emigrants in the course of a year, what is its net annual percentage growth?  
Net annual percentage growth =  $\frac{\text{Growth rate}}{\text{Total popn}} \times 100 = \underline{\hspace{2cm}}$
2. The current global human population is about 6.1 billion and is growing at an annual rate of 1.35 percent. If world population were to grow at this rate for the next year, approximately how many people would be added?  
Current population x growth rate = number of people being added
3. If a population grows at a rate of approximately 5 percent per year, how many years are required for the population to double?  
Rule of 70... time it takes population to double =  $\frac{70}{\text{Growth rate}} = \underline{\hspace{2cm}}$
4. If a population of 100,000 experiences 2,000 births, 1,600 deaths, 200 immigrants, and 100 emigrants in the course of one year, what is its net annual percentage growth rate?
5. The current US population is approximately 300 million. If its annual growth rate is 1.0% and continues to grow at this rate for one year, approximately how many people will be added?
6. If the % annual growth rate is 8%, what is the doubling time in years?
7. If the doubling time is 7 years, what is the % annual growth rate?
8. A bank advertises doubling your investment money in 12.3 years. What percent simple interest will it be paying?
9. The world population grows at a rate of 1.2% annually.
  - a. What is the doubling time for the world population?
  - b. What would be the doubling time if the growth rate is cut to 1%?
  - c. Assuming the growth rate stays constant (at 1.2%), how many doublings would take place in a 500-year period?
  - d. What would the world's population be in 500 years if there are currently 7 billion people? (Think: one doubling is how long? So after that time you'd have 14 billion. Then you'd have 28 billion after a second doubling time had passed... etc.)
  - e. If there are currently 7 billion people in the world, what year will there be 14 billion?
  - f. Extension Opportunity: Assuming you live to your life expectancy (78.7 years), what will the population be in the year you die?



## Population Math

### Formulas:

$$\text{Population Density: } \frac{\text{total population}}{\text{total area}}$$

$$\text{Birth Rate (as a \%): } \left( \frac{\text{total births}}{\text{total population}} \right) \times 100$$

$$\text{Birth Rate (per 1000): } \left( \frac{\text{total births}}{\text{total population}} \right) \div 1000$$

$$\text{Death Rate (as a \%): } \left( \frac{\text{total deaths}}{\text{total population}} \right) \times 100$$

$$\text{Death Rate (per 1000): } \left( \frac{\text{total deaths}}{\text{total population}} \right) \div 1000$$

$$\text{Crude Birth Rate: } \left( \frac{\text{total \# births}}{\text{total population}} \right) \times 1000$$

$$\text{Crude Death Rate: } \left( \frac{\text{total \# death}}{\text{total population}} \right) \times 1000$$

$$\text{Population Change: } \frac{(\text{births} + \text{immigration}) - (\text{deaths} + \text{emigration})}{\text{total population}}$$

$$\text{Population Growth Rate: } \left( \frac{(\text{births} + \text{immigration}) - (\text{deaths} + \text{emigration})}{\text{total population}} \right) \times 100$$

$$\text{Doubling Time: } \frac{70}{\% \text{ growth rate}} = \text{years to double}$$

$$\text{Rate of Change: } \frac{(\text{new-old})}{\text{old}}$$

$$\text{Percent Change: } \frac{(\text{new-old})}{\text{old}} \times 100$$

Natural Rate of Population Increase: births-deaths

**Population growth equations to know for the AP exam**

Simple growth rate of a population	$N_1 = N_0 + B - D + I - E$
Intrinsic Rate (r) of increase for population growth	$r = B - D$
Rate of change of population size	$\frac{dN}{dt} = rN \quad dN = N_1 - N_0$
Net growth rate of a population ( $R_0$ )	$R_0 = \frac{N_1}{N_0}$
Doubling time for a population (rule of 70)	$Dt = 70 / R_0$
Annual % rate of natural population change	$\% = \frac{(B - D)}{1,000} \times 100\%$

**Practice Problems**

- In 1950, the population of a small suburb in Los Angeles, California, was 20,000. The birth rate was measured at 25 per 1000 population per year, while the death rate was measured at 7 per 1000 population per year. Immigration was measured at 600 per year while emigration was measured at 200 per year. Calculate the population size in 1951.  
*20,700*
- Calculate the rate of increase for population growth for the preceding problem.
- Calculate the rate of change for the population in the first example.  
*18 per 1000*
- Calculate the net growth rate of the population described in the first example. A population that was not growing would have a net growth rate of 1. A population that was doubling would have a net growth rate of 2, and so on.  
*1.038*
- A population had a growth rate of 1.7 in 1995. In what year would the population be double its current value? Assume all other factors are constant.  
*4 yrs*
- In 2,000, the world birth rate was 22 births per 1000 people. The world death rate was 9 deaths per 1000 people. Calculate the percent world growth rate.  
*1995 + 41 = 2036*
- Uganda has a crude birth rate of 48 and a crude death rate of 18.4. Calculate the percent population growth rate for Uganda.  
*2.9%*



Name: \_\_\_\_\_ Period: \_\_\_\_\_ Date: \_\_\_\_\_

## Calculating Population Growth Rate and Doubling Time

There are four variables that influence the growth rate of the human population. Crude birth rate (CBR) and immigration are inputs and increase the population. Crude death rate (CDR) and emigration are outputs and decrease the population. The human population can be studied as a system of inputs and outputs; if the net input is greater than output, then the population expands. All four variables are usually given per 1,000 which will simplify calculations quite a bit. Let's take a look at a few equations.

**Global Population Growth Rate:** Since people do not immigrate or emigrate from the planet, we do not include those variables in calculating the growth rate of the world population. The equation we use is:

$$\text{Global Population growth rate} = \frac{(\text{CBR} - \text{CDR})}{10}$$

**Sample problem:** Worldwide, there were 20 births and 8 deaths per 1,000 in 2009. Calculate the growth rate of the world in 2009.

$$\frac{20 - 8}{10} = \frac{12}{10} = 1.2$$

**Growth rate of a single nation or region:** Since people do migrate in and out of nations, we do include immigration and emigration into the equation.

$$\text{Growth Rate for a nation or region} = \frac{(\text{CBR} + \text{I}) - (\text{CDR} + \text{E})}{10}$$

**Sample problem:** The tiny country of Fremont has a population of 100,000 people. In 2009, there were 2,000 births, 500 deaths, 200 emigrants, and 100 immigrants. What is the population growth rate (r) for 2009?

$$\frac{(2000 + 100) - (500 + 200)}{10} = \frac{2100 - 700}{10} = \frac{1400}{100,000} = 0.14$$

**Rule of 70 or Doubling time:** If we know the growth rate (r) of a population and assume that it is constant, we can calculate the number of years it will take for the population to double, known as the doubling rate. Because growth rates may change in future years, we can never calculate a country's doubling time with certainty. We can say that a population will double in a certain number of years if the growth rate of the population remains constant. The doubling time can be approximated mathematically using the formula called the rule of 70.

$$\text{Doubling time (in years)} = \frac{70}{\text{Growth rate (r)}}$$

**Sample problem:** The small town of West Fremont has a population of 50,000. If the growth rate of West Fremont is 2%, then how long will it take for the population of West Fremont to double?

$$\frac{70}{2} = 35 \text{ yrs}$$

1. North Fremont's population growth rate is 5%. What is the doubling time for North Fremont?

$$\frac{70}{5} = 14$$

2. New Fremont had a birthrate of 12 per 1,000 in 2010 and a death rate of 9 per 1,000.

- a. Calculate the growth rate of New Fremont.

$$\frac{12-9}{10} = \frac{3}{10} = .3\%$$

- b. If the current population of New Fremont is 150,000, how long will it take the country to double its population using the current growth rate? (Round to the nearest whole number.)

$$\frac{70}{.3} = 223 \text{ yrs}$$

3. Central Fremont has a crude birth rate of 24 per 1,000 and a crude death rate of 8 per 1,000.

- a. What is the natural annual increase of Central Fremont?

~~$$\frac{70}{1.6} = 44 \text{ yrs}$$~~

$$\frac{24-8}{10} =$$

- b. At this rate of increase, how long will it take Central Fremont's population of 35,000 to double? (Round to the nearest whole number.)

$$\frac{70}{1.6} = 44 \text{ yrs}$$

4. In 2010, the crude birth rate in Lower Fremont was 25 and the crude death rate was 11. If the population was 15,000 in 2010, and the population growth rate remains constant, when will the population reach 30,000?

$$\frac{25-11}{10} = \frac{14}{10} = 1.4\% \quad \frac{70}{1.4} = 50 \text{ yrs}$$

$$2010 + 50 = 2060$$

5. In 2010, East Fremont had a population of 10 million people, a birth rate of 7.2%, and a death rate of 2.2%. If the birth and death rates remain constant, in what year will the population will be close to 40 million people?

$$7.2 - 2.2 = 5\% \quad \frac{70}{5} = 14 \times 2 = 28 \text{ yrs}$$

$$2010 + 28 = 2038$$

6. In 2010, the population of Fremontville was 6 million and growing at a rate of 1.4% / year. If the rate of population growth remains constant, in what year will the population reach 24 million people?

$$\frac{70}{1.4} = 50 \times 2 = 100$$

$$2010 + 100 = 2110$$

NAME key

Period \_\_\_\_\_

## Population – Math Practice Worksheet

1. One thousand two hundred and seventy deer are living on an island that is eight hundred and thirty square kilometers in size. What is the population density of the deer per square kilometer?

$$\frac{1270 \text{ deer}}{830 \text{ km}^2} = 1.53 \text{ deer/km}^2$$

2. A city with 53,340 people has 876 births. What is the birth rate (percentage and crude birth)?

$$\frac{876}{53,340} = 0.016 \times 100 = 1.6\%$$

3. Another city (population 8,436) experiences 120 deaths. What is the death rate (percentage and crude death)?

$$\frac{120}{8,436} = 0.014 \times 100 = 1.4\%$$

4. A village of 23,473 people has 2,342 births and 473 deaths. What is the growth rate for this village?

$$\frac{2,342 - 473}{23,473} = 0.0796 \times 100 = 7.97\%$$

5. A small country of 744,785 people has 44,678 immigrants and 12,567 emigrants. They also experience 15,898 deaths and 35,665 births. What is the growth rate of this small country?

$$\frac{(35,665 + 44,678) - (15,898 + 12,567)}{744,785} = \frac{51,878}{744,785} = 0.0697 \times 100 = 6.97\%$$

How many years will it take for this country to double its population?

$$\frac{70}{6.97} \approx 10 \text{ years}$$

6. If a country were doubling its population every 35 years, what would its growth rate be?

$$\frac{70}{x} = 35 \quad \frac{70}{35} = x \quad x = 2 \text{ years}$$

7. At the end of 2002, there were 1,284,530,000 people living in China. China is the third largest country in the world with an area of 9.6 million square kilometers. What is the population density of China?

$$\frac{1,284,530,000 \text{ ppl}}{9.6 \times 10^6 \text{ km}^2} = 133.8 \text{ ppl/km}^2$$

In 2002, 16.47 million babies were born in China. What was the birth rate (percentage and crude birth)?

$$\frac{16.47 \times 10^6}{1,284,530,000} = 0.0128 \times 100 = 1.28\%$$

In 2002, 8.21 million people died in China. What was the death rate (percentage and crude death)?

$$\frac{8.21 \times 10^6}{1284530000} = 0.0064 \times 100 = \text{0.64\%}$$

What was the total overall growth rate of China's population in 2002?

$$1.28 - 0.64 = 0.64\%$$

Using the rate from the previous question, how many years will it take for China's population to double?

$$\frac{70}{.64} = 26.9 \sim 27 \text{ years}$$

8. In 2000, there were 30,750,087 people living in Canada, which has a total area of 9,984,670 km<sup>2</sup>. What was the population density of Canada?

$$\frac{30750087 \text{ ppl}}{9984670 \text{ km}^2} = 3.08 \text{ ppl/km}^2$$

In 2000, there were 335,500 births in Canada. What was the crude birth rate?

$$\frac{335500}{30750087} = 0.011 \times 100 = 1.1\%$$

In Canada during the same year there were 225,500 deaths, 205,711 immigrants and 41,142 emigrants. What was the growth rate for Canada, expressed as a percentage?

$$\frac{335500 + 205711 - 41142 - 225500}{30750087} = \frac{274569}{30750087} = 0.0089 \times 100 = 0.89\%$$

How many years will it take for Canada's population to double?

$$\frac{70}{.89} = 78.6 \sim 79 \text{ years}$$

9. Which of these countries will have the largest population in 100 years? Assume that the population growth rate will remain the same.

- A. Madagascar with 15.2 million people at 3.2% growth rate
- B. Columbia with 38.0 million at 2.1% growth rate
- C. Sudan with 28.9 million people and 3.0% growth rate
- D. France with 58.4 million and 0.3% growth rate
- E. Cote d'Ivoire with 14.7 million people and 3.5% growth rate

$$\begin{aligned} (15.2 \times 10^6)(1.032^{100}) &\approx 3.5 \times 10^8 \\ (38 \times 10^6)(1.021^{100}) &\approx 2.4 \times 10^8 \\ (28.9 \times 10^6)(1.03^{100}) &\approx 5.5 \times 10^8 \\ (58.4 \times 10^6)(1.003^{100}) &\approx 7.37 \times 10^7 \\ (14.7 \times 10^6)(1.035^{100}) &\approx 3.43 \times 10^8 \end{aligned}$$